Cycle 2: Why Does It Matter?

Part 1

Lesson 2.2 It's All Relative

Skills:

1. $-\left|-15\right| = -15$

2. -2

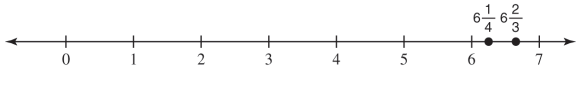
Concepts and Applications:

3. a. Negative; $-\$10,000$
   b. Negative; -400

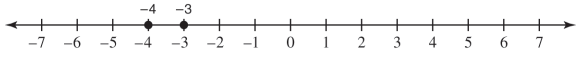
4. a. Loss of 20 yards
   b. Loss of 120.62 points for the day

5.

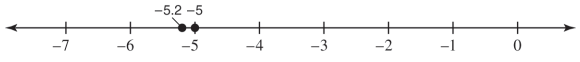
6. a. $6\frac{1}{4} < 6\frac{2}{3}$



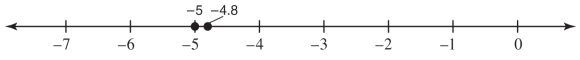
   b. $-3 > -4$



   c. $-5 > -5.2$



   d. $-5 < -4.8$



7. The student's analogy with money is actually correct. However, that does not mean that -5 is bigger than -4. The bigger the debt, the more the person is "in the hole" and the less money he or she has. Therefore, -5 is less than -4.

8. a. -7
   b. 7
   c. 7, 7
   d. *a*

Lesson 2.3 Sign and Size, Part 1

Skills:

1. a. $5.2 - (-1.1) = 5.2 + 1.1 = 6.3$

   b. $-13 + (-31) = -(13 + 31) = -44$

Concepts and Applications:

2. Answers will vary depending on the year. Possible answer: $2013-(-500)=2013+500=2513$ years

3. $-\$10-\$25=-(\$10+\$25)=-\$35$
She will owe him \$35 in total.

4. $-\$11,500+\$20,000=\$20,000-\$11,500=\$8,500$
She will have \$8,500 after the debt is paid.

5. $6-15+12=3$
The team has already gained 3 yards on the first three downs. They will need $10-3=7$ more yards on the fourth down to have a 10-yard gain (and get a first down).

Lesson 2.4 Sign and Size, Part 2

Skills:

1. a. $(0)(-15)=0$

   b. $\dfrac{-15}{3}=-5$

2. $(-1)(-2)(-7)=(2)(-7)=-14$

Concepts and Applications:

3. a. Multiplying two negative numbers results in a positive number. For example, $(-4)(-4)=16$.
   Adding two negative numbers results in a negative number. For example, $(-4)+(-4)=-8$

   b. No. Two negative numbers can result in a positive, negative, or zero result depending on the operation.

4. Answers will vary. Possible answer: Two wrongs don't make a right.

5.

	Problem with parentheses	Answer	Problem without Parentheses	New Result	Do parentheses matter?
a.	$2(-4)$	-8	$2-4$	-2	Yes
b.	$(-2)+(-4)$	-6	$-2+-4$	-6	No
c.	$2-(4)$	-2	$2-4$	-2	No
d.	$(-2)(-4)$	8	$-2-4$	-6	Yes

6. a. $6\times ?=0$ $0\div 6=0$
   $?=0$

   b. $0\times ?=6$ $6\div 0$ is undefined
   no such number

c. Zero divided by a number (other than zero) is zero. A number cannot be divided by zero.

7. a. Positive
b. It depends. Possible examples: $3+(-7)=-4$, $-3+7=4$
c. Negative
d. Positive
e. Positive
f. Negative
g. Negative

8. Answers will vary.

Possible calculation with a negative number:

-3
$-3-1=-4$
$-4\cdot 3=-12$
$-12+6=-6$
$-6\div 3=-2$
$-2-(-3)=-2+3=1$

Possible calculation with a fraction:

$\frac{2}{3}$
$\frac{2}{3}-1=-\frac{1}{3}$
$-\frac{1}{3}\cdot 3=-1$
$-1+6=5$
$5\div 3=\frac{5}{3}$
$\frac{5}{3}-\frac{2}{3}=1$

Yes. The result is always 1.

Lesson 2.5 An Ounce of Prevention

Skills:

1. $\frac{-2+7+8+4+(-1)+(-10)+1}{7}=\frac{7}{7}=1$

2. Current average: $\frac{75+74+71}{3}=\frac{220}{3}\approx 73.3$

To average an 80, $\frac{\text{point total}}{4}=80$. So her point total must be 320 points. She needs $320-220=100$ on the last test to have a B average.

Concepts and Applications:

3. Answers will vary. Any 3 scores, between 0 and 10, which add to 21 will work.
Possible answers: 7, 7, 7; 6, 7, 8; 10, 10, 1; 9, 9, 3; 5, 6, 10

4. Answers will vary. It's often easy to gain weight, but hard to lose it. It is easy to get in debt, but hard to get out of it. Addition is usually easier than subtraction. Multiplication is easier than division.

5. To split the check evenly, the total bill must be divided by five.
You would each pay $\frac{\$155}{5}=\$31$. This amount also represents the average of the individual bills.

6. a. $\frac{32+36+38+39+42+43+44+47+49+50}{10} = \frac{420}{10} = 42$

The mean income for Group 1 is \$42,000/year

$\frac{32+36+38+39+42+43+44+47+49+150}{10} = \frac{520}{10} = 52$

The mean income for Group 2 is \$52,000/year

b. The mean is affected by one extreme value in a data set. When the last income was changed from \$50,000 to \$150,000, the mean increased by \$10,000.

c. No. For Group 2, a mean of \$52,000/year is not very representative of the typical income since only one of the ten incomes is higher than that.

7. a. Answers will vary. Possible answer: $40, 45, 50, 55, 60$

b. $50, 55, 60, 65, 70$

c. New mean: 60

d. When 10 was added to each score, the mean also increased by 10.

e. To increase the mean by 8 points, the teacher should add 8 points to each student's test score.

8. a. $\frac{50+100}{2} = \frac{150}{2} = 75$

b. $\frac{75+75}{2} = \frac{150}{2} = 75$

c. The mean does not tell us whether the two test scores are close to the mean of 75 or more spread out. The mean only tells us about the center, not the spread of the data.

9. a. $\frac{7+13+6+14+5+15}{6} = \frac{60}{6} = 10$

b. The mean is the balancing point for the data. If we think of the number line as a teeter-totter, the mean of 10 is where the data values will balance.

10. a. The average height should be in inches, not square inches. Height is a linear measurement, not an area.

b. The average square footage should be in square feet, not feet. Square footage is a measurement of area, not length.

Lesson 2.6 Measure Up

Skills:

1. a.

$$\begin{aligned}(14ab)^3(ab) &= 14^3a^3b^3(ab)\\ &= 2,744a^{3+1}b^{3+1}\\ &= 2,744a^4b^4\end{aligned}$$

b. $15^0(3x^2)^3 = 1\cdot 3^3x^{2\cdot 3} = 27x^6$

c. $\frac{4m^9}{8m^3n} = \frac{4m^{9-3}}{8n} = \frac{m^6}{2n}$

2. a. $C = 2\pi r = 2\pi(24 \text{ in.}) \approx 151 \text{ in.}$

b. $A = \pi r^2 = \pi(2 \text{ ft})^2 \approx 13 \text{ ft}^2$

Concepts and Applications:

3.

$$V = \frac{m}{d}$$
$$= \frac{g}{g / c^3}$$
$$= g \div \frac{g}{c^3}$$
$$= g \cdot \frac{c^3}{g}$$
$$= c^3$$

The volume will be in cubic centimeters.

4.

$$D = \frac{v^2}{g} \cdot 0.87$$
$$= \frac{(\text{m/sec})^2}{\text{m/sec}^2}$$
$$= \frac{\text{m}^2/\text{sec}^2}{\text{m/sec}^2}$$
$$= \frac{\text{m}^2}{\text{sec}^2} \div \frac{\text{m}}{\text{sec}^2}$$
$$= \frac{\text{m}^2}{\text{sec}^2} \cdot \frac{\text{sec}^2}{\text{m}}$$
$$= \text{m}$$

The distance will be in meters.

5.

$$\begin{aligned} T &= 2\pi\sqrt{\frac{L}{g}} \\ &= \sqrt{\frac{\text{ft}}{\text{ft}/\text{sec}^2}} \\ &= \sqrt{\text{ft} \div \frac{\text{ft}}{\text{sec}^2}} \\ &= \sqrt{\text{ft} \cdot \frac{\text{sec}^2}{\text{ft}}} \\ &= \sqrt{\text{sec}^2} \\ &= \text{sec} \end{aligned}$$

The time will be in seconds.

Lesson 2.7 Count Up

Skills:

1. No, they are not like terms. They do not have the same exponents on the *x* factors and they do not have the same exponents on the *y* factors.

2.

$$\begin{aligned} -11x^2 + 11xy + 22xy - 12x^2 + 2 &= (11+22)xy + (-11-12)x^2 + 2 \\ &= 33xy - 23x^2 + 2 \end{aligned}$$

3.

$$\begin{aligned} (-11x^2)(11xy)(22xy)(-12x^2)(2) &= 63,888x^{2+1+1+2}y^{1+1} \\ &= 63,888x^6y^2 \end{aligned}$$

4. The polynomial is a trinomial since it has three terms.

Concepts and Applications:

5. a. The student appears to have added $5x + 6y$ to get $11xy$, but they are not like terms and should not be combined. Instead, the *xy* terms should be combined and the x term and y term should be left separate.

$$\begin{aligned} -3xy + 5x + 6y - 8xy &= (-3xy - 8xy) + 5x + 6y \\ &= -11xy + 5x + 6y \end{aligned}$$

b. The expression still has like terms ($15x^2$ and $-18x^2$) that should be combined.

$$\begin{aligned} 15x^2 + 10x - 21x - 18x^2 &= (15-18)x^2 + (10-21)x \\ &= -3x^2 - 11x \end{aligned}$$

6. Answers will vary. Possible answer:
Expression that contains like terms: $5x + 6y - 2x$

Expression that does not contain like terms: $5x + 6y - 2xy$

7.

$$\begin{aligned} S &= \pi r^2 + \pi rl \\ &= \pi(2 \text{ in.})^2 + \pi(2 \text{ in.})(4 \text{ in.}) \\ &= \pi(4 \text{ in.}^2) + \pi(8 \text{ in.}^2) \\ &= (4+8)\pi \text{ in.}^2 \\ &= 12\pi \text{ in.}^2 \\ &\approx 37.7 \text{ in.}^2 \end{aligned}$$

Cycle 2 Part 1 Recap

Skills:

1. a. $(5x^5)^2 = 5^2(x^5)^2 = 25x^{10}$

   b. $\dfrac{50x^{50}}{5x^5} = 10x^{50-5} = 10x^{45}$

   c. $5x^4 + 5x^4 = 10x^4$

   d. $5x^4 \cdot 5x^4 = 25x^{4+4} = 25x^8$

   e. $-5^0 = -1 \cdot 5^0 = -1 \cdot 1 = -1$

   f. $(-5)^0 = 1$

Concepts and Applications

2. a. Yes; The sum of negative numbers will always be negative. The average will then be a negative sum divided by a positive number, which will always be negative.

   b. Answers will vary. Possible answer:
   $0, 0, 0, 0, 0$
   $-3, -2, -1, 1, 2, 3$

3.

$$\begin{aligned} -\$50(4) - \$25(3) - \$100 + \$75(2) &= -\$200 - \$75 - \$100 + \$150 \\ &= -\$375 + \$150 \\ &= -\$225 \end{aligned}$$

Your net debt is \$225.

Cycle 2: Why Does It Matter?

Part 2

Lesson 2.9 Order Up

Skills:

1. a.

$$\frac{5}{9}(14-32)=\frac{5}{9}(-18)$$
$$=\frac{5}{\cancel{9}_{1}}\cdot\frac{\cancel{-18}^{-2}}{1}$$
$$=-10$$

b.

$$20\cdot\frac{9}{5}+32=\frac{\cancel{20}^{4}}{1}\cdot\frac{9}{\cancel{5}_{1}}+32$$
$$=36+32$$
$$=68$$

2. a.

$$\frac{5x}{y^2}=\frac{5(-2)}{(-4)^2}$$
$$=\frac{-10}{-4\cdot-4}$$
$$=\frac{-10}{16}$$
$$=-\frac{5}{8}$$

$$\frac{5x}{y^2}=\frac{5(0)}{(-4)^2}$$
$$=\frac{0}{(-4)^2}$$
$$=0$$

$$\frac{5x}{y^2}=\frac{5(-4)}{(0)^2}$$
$$=\frac{-20}{0}$$
undefined

b.

$$\text{BMI}=\frac{(\text{weight in pounds})(703)}{(\text{height in inches})^2}$$
$$=\frac{(150\text{ lb})(703)}{(72\text{ in.})^2}$$
$$=\frac{105,450\text{ lb}}{5,184\text{ in.}^2}$$
$$\approx 20.34\text{ lb/in.}^2$$

Concepts and Applications:

3. Answers will vary based on the calculator used.

4. a. Knee height = $16\text{ in.}\cdot 2.54\text{ cm/in.} = 40.64\text{ cm}$

Height in centimeters $= 84.88-(0.24\times 80)+(1.83\times 40.64)\approx 140.05\text{ cm}$

Height in inches $= 140.05\text{ cm}\div 2.54\text{ cm/in.}\approx 55.14\text{ in.}$

b. Answers will vary.

5. Expression d best illustrates the process to make cupcakes. The dry ingredients are combined and the wet ingredients are combined. And then the total is divided into 24 cupcakes.

6. a. The multiplier is 1.097.
$1.0975(\$45) \approx \$49.39$

b. The multiplier is 0.80.
$0.80(\$75) = \$60$

Lesson 2.10 Does Order Matter?

Skills:

1. a. $(y + x) + z$ or $z + (x + y)$

b. $x + (y + z)$

2. a. $(yx)z$ or $z(xy)$

b. $x(yz)$

Concepts and Applications:

3. a. Answers will vary. Possible answer:
$6 \div 2 = 3$ but $2 \div 6 = \frac{1}{3}$

b. "Divide 5 by 25" implies $5 \div 25$ when the intended problem is $25 \div 5$. You could say, "Divide 25 by 5" or "Divide 5 into 25." How you verbalize the problem matters since it indicates the order of the numbers in the division problem.

4. a. $2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 = 16$

b. Switch the 8 and the 2.
$8 \times 2 = 2 \times 8 = 8 + 8 = 16$

c. 2×8 is easier to do with repeated addition because there are fewer numbers to add and keep track of in your head.

5. a. Your friend's grade would be represented by $x - 5$ since it is 5 points lower than yours.

b. Your grade would be represented by $x + 5$ or $5 + x$ since it is 5 points higher than his.

6. Group all the positive numbers together and all the negative numbers together and then combine those results.

7. a. Commutative Property
b. Associative Property

8. a. $\frac{2}{3}\cdot\frac{15}{14}=\frac{30}{42}=\frac{30\div 6}{42\div 6}=\frac{5}{7}$

b. $\frac{2}{3}\cdot\frac{15}{14}=\frac{\cancel{2}^{1}}{\cancel{3}_{1}}\cdot\frac{\cancel{15}^{5}}{\cancel{14}_{7}}=\frac{5}{7}$

c. The two operations commute. You can multiply first then simplify or simplify first then multiply. You will get the same answer either way.

Lesson 2.11 Fair Share

Skills:

1. a. $-4(x-16x^2)=-4x-4\cdot -16x^2=-4x+64x^2$

b.
$$\begin{aligned}(2x-1)(x+1)&=2x(x+1)-1(x+1)\\&=2x^2+2x-x-1\\&=2x^2+x-1\end{aligned}$$

2.
$$\begin{aligned}7\times 22&=7(20+2)\\&=7\times 20+7\times 2\\&=140+14\\&=154\end{aligned}$$

Concepts and Applications:

3. a.
$$\begin{aligned}2\frac{1}{2}\times 3\frac{2}{3}&=\frac{5}{2}\times\frac{11}{3}\\&=\frac{55}{6}\\&=9\frac{1}{6}\end{aligned}$$

b.

$$2\frac{1}{2}\times 3\frac{2}{3}=\left(2+\frac{1}{2}\right)\times\left(3+\frac{2}{3}\right)$$
$$=2\left(3+\frac{2}{3}\right)+\frac{1}{2}\left(3+\frac{2}{3}\right)$$
$$=6+\frac{4}{3}+\frac{3}{2}+\frac{1}{3}$$
$$=6+\frac{5}{3}+\frac{3}{2}$$
$$=6+\frac{10}{6}+\frac{9}{6}$$
$$=6+\frac{19}{6}$$
$$=6+3\frac{1}{6}$$
$$=9\frac{1}{6}$$

Yes

c. No

4. a. Distributive Property

b. Commutative Property

c. $6\frac{2}{3}-3\frac{1}{3}=\frac{20}{3}-\frac{10}{3}=\frac{10}{3}=3\frac{1}{3}$

d. Method 1 (student's nontraditional way):

$$2\frac{1}{4}-1\frac{3}{4}=2+\frac{1}{4}-1-\frac{3}{4}$$
$$=1-\frac{2}{4}$$
$$=\frac{2}{4}$$
$$=\frac{1}{2}$$

Method 2 (traditional approach):

$$2\frac{1}{4}-1\frac{3}{4}=\frac{9}{4}-\frac{7}{4}$$
$$=\frac{2}{4}$$
$$=\frac{1}{2}$$

5. a.

$$\begin{aligned}(a-b)(a+b) &= (\mathrm{a}-\mathrm{b})\,\mathrm{a}+(\mathrm{a}-\mathrm{b})\,\mathrm{b} \\ &= \mathrm{a}^2 - ba + ab - b^2 \\ &= a^2 - b^2\end{aligned}$$

b.

$$\begin{aligned}49\times 51 &= (50-1)(50+1) \\ &= 50^2 - 1^2 \\ &= 2,500 - 1 \\ &= 2,499\end{aligned}$$

c.

$$\begin{aligned}72\times 68 &= (70+2)(70-2) \\ &= (70-2)(70+2) \\ &= 70^2 - 2^2 \\ &= 4,900 - 4 \\ &= 4,896\end{aligned}$$

Commutative Property for Multiplication

6. Each output is one less than three times the input value.

Input	Output
0	-1
2	5
5	14
11	32
100	299
n	$3n-1$

Lesson 2.12 Seat Yourself

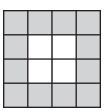
Skills:

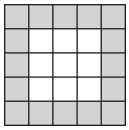
1. $5(x+8)-4(x-1)+6 = 5x+40-4x+4+6 = x+50$

2. $8-3(2-n)-17 = 8-6+3n-17 = 3n-15$

Concepts and Applications:

3. a.





b. One way to think about this is that there are 4 corner tables that each seat 2 people. All the other tables ($n-4$ of them) seat 1 person. So there are $4\cdot 2+(n-4)\cdot 1=8+n-4=n+4$ seats for *n* tables.

c.

Number of tables	Number of chairs
4	8
8	12
12	16
16	20
n	$n+4$

You get the same general result, $n+4$, as was achieved by using the physical situation in part b.

d. Yes. The number of tables in this arrangement must be a multiple of 4, and 20 is a multiple of 4. If 20 tables were used, 24 people could be seated.

e.

$$n+4=36$$
$$n=32$$

32 tables would be needed to seat 36 people.

4. a.

b. $\frac{3+19}{2}=\frac{22}{2}=11$

You will meet at mile marker 11.

c. $B-A$

d. $\frac{1}{2}(B-A)$

e. $A+\frac{1}{2}(B-A)=A+\frac{1}{2}B-\frac{1}{2}A=\frac{1}{2}A+\frac{1}{2}B$

f. $B-\frac{1}{2}(B-A)=B-\frac{1}{2}B+\frac{1}{2}A=\frac{1}{2}A+\frac{1}{2}B$

g. $\frac{A+B}{2}=\frac{1}{2}(A+B)=\frac{1}{2}A+\frac{1}{2}B$

Yes, they are equivalent.

Lesson 2.13 Punt, Pass, Kick

Skills:

1.

$$(5 \text{ in.})^2 + (5 \text{ in.})^2 = \text{hyp}^2$$
$$25 \text{ in.}^2 + 25 \text{ in.}^2 = \text{hyp}^2$$
$$50 \text{ in.}^2 = \text{hyp}^2$$
$$\text{hyp} = \sqrt{50 \text{ in.}^2}$$
$$\text{hyp} \approx 7.1 \text{ in.}$$

2.

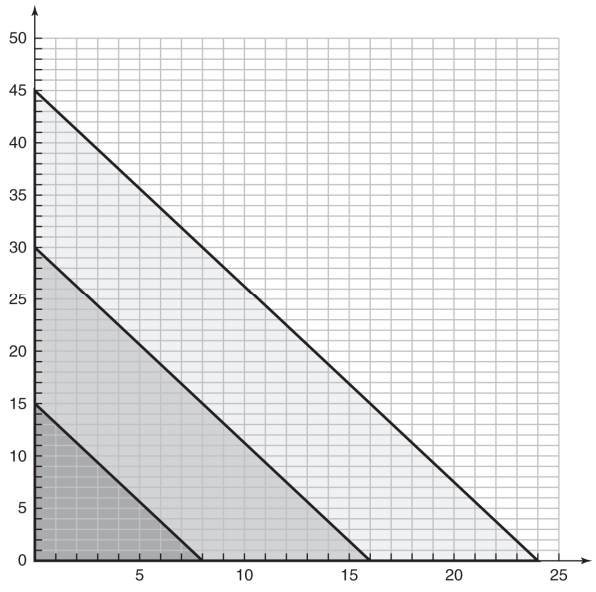
$$(4 \text{ ft})^2 + (\text{leg})^2 = (10 \text{ ft})^2$$
$$16 \text{ ft}^2 + (\text{leg})^2 = 100 \text{ ft}^2$$
$$(\text{leg})^2 = 84 \text{ ft}^2$$
$$\text{leg} \approx 9.2 \text{ ft}$$

Concepts and Applications:

3. a. All are whole numbers and $8^2 + 15^2 = 289 = 17^2$
   All are whole numbers and $16^2 + 30^2 = 1{,}156 = 34^2$

   b. 24, 45, 51

   c.



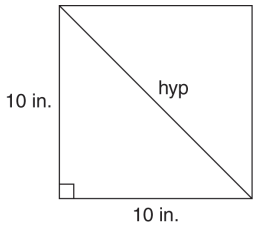
   d. The triangles are similar.

4. a. No. Since $3 + 4 = 7$, the lengths do not make a triangle. In order to make a triangle the sum of the lengths of the shorter sides must be greater than the length of the longest side.

b. Yes. Since $3+4>6$, the lengths do make a triangle.
No. Since $3^2+4^2 \neq 6^2$, the lengths do not make a right triangle.

c. Answers will vary. Some possible answers: $4 \text{ ft}, 5 \text{ ft}, 6 \text{ ft}$; $12 \text{ in.}, 18 \text{ in.}, 20 \text{ in.}$

5.

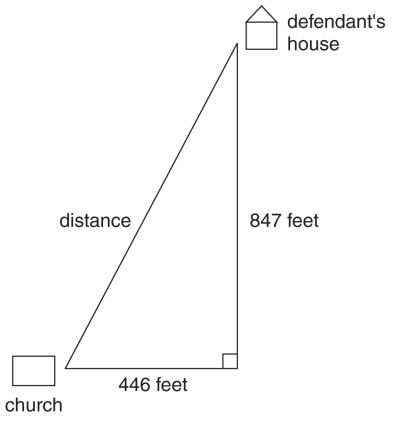


If the area of the square is 100 square inches, then each side is 10 inches long.

$$(10 \text{ in.})^2 + (10 \text{ in.})^2 = \text{hyp}^2$$
$$100 \text{ in.}^2 + 100 \text{ in.}^2 = \text{hyp}^2$$
$$200 \text{ in.}^2 = \text{hyp}^2$$
$$\text{hyp} = \sqrt{200 \text{ in.}^2}$$
$$\text{hyp} \approx 14.1 \text{ in.}$$

So the diagonal of the square is about 14.1 inches long.

6.



a.

$$847^2 + 446^2 = \text{distance}^2$$
$$717,409 + 198,916 = \text{distance}^2$$
$$916,325 = \text{distance}^2$$
$$\sqrt{916,325} = \text{distance}$$
$$\text{distance} \approx 957 \text{ft}$$

b. Yes. The crime was committed about 957 feet from a church, making it a felony.

Lesson 2.14 Ramp Up

Skills:

1.
$$\begin{aligned} m &= \frac{-3-11}{2--8} \\ &= \frac{-3+-11}{2+8} \\ &= \frac{-14}{10} \\ &= -\frac{7}{5} \end{aligned}$$

2. The graph goes through the points $(0,3)$ and $(5,0)$

$$m = \frac{0-3}{5-0} = \frac{-3}{5} = -\frac{3}{5}$$

Concepts and Applications:

3. Every time x increases by 1, y decreases by 5. So $m = \dfrac{\text{change in } y}{\text{change in } x} = \dfrac{-5}{1} = -5$.

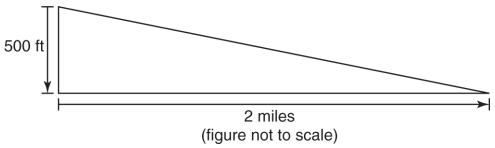
4. a. $$m = \frac{\text{rise}}{\text{run}} = \frac{70 \text{ inches}}{110 \text{ inches}} = \frac{7}{11}$$

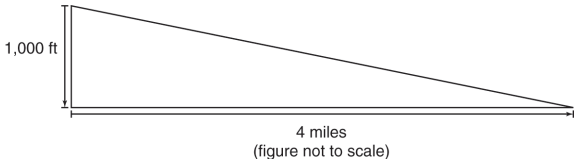
b. $$m = \frac{\text{rise}}{\text{run}} = \frac{7 \text{ inches}}{11 \text{ inches}} = \frac{7}{11}$$

c.
$$\begin{aligned} (70 \text{ in.})^2 + (110 \text{ in.})^2 &= \text{hyp}^2 \\ 4{,}900 \text{ in.}^2 + 12{,}100 \text{ in.}^2 &= \text{hyp}^2 \\ 17{,}000 \text{ in.}^2 &= \text{hyp}^2 \\ \text{hyp} &= \sqrt{17{,}000 \text{ in.}^2} \\ \text{hyp} &\approx 130 \text{ in.} = 10 \text{ ft, } 10 \text{ in.} \end{aligned}$$

5. For fractions with a numerator of 1, the larger the denominator, the smaller the fraction. So $\dfrac{1}{20} < \dfrac{1}{16} < \dfrac{1}{12}$. The ramp with a slope of 1:12 is the steepest because it has the largest slope of the three and the shortest "run" for the rise of 1. The ramp with a slope of 1:20 is the flattest because it has the smallest slope of the three and the longest "run" for a rise of 1.

6. a.





b. $$m = \frac{-250 \text{ ft}}{1 \text{ mi}} = -250 \text{ ft/mi}$$

$$m = \frac{-500 \text{ ft}}{2 \text{ mi}} = -250 \text{ ft/mi}$$

$$m = \frac{-1,000 \text{ ft}}{4 \text{ mi}} = -250 \text{ ft/mi}$$

We get the same slope regardless of which triangle is used.

c. In order to write the slope as a percent, the units on the rise and run need to be the same.

$$m = \frac{-250 \text{ ft}}{5,280 \text{ ft}} = -0.047 = -4.7\%$$

Yes. The truck can drive safely on these roads since the slope is less than 6%,

7. a. The student put the change in x on the top of the slope fraction and the change in y on the bottom of the slope fraction.

b. The student subtracted in a different order in the numerator vs. the denominator of the slope fraction.

c. The student did not simplify the final fraction to $\frac{3}{2}$.

8. First triangle:

$$(250 \text{ ft})^2 + (5,280 \text{ ft})^2 = \text{hyp}^2$$
$$62,500 \text{ ft}^2 + 27,878,400 \text{ ft}^2 = \text{hyp}^2$$
$$27,940,900 \text{ ft}^2 = \text{hyp}^2$$
$$\text{hyp} = \sqrt{27,940,900 \text{ ft}^2}$$
$$\text{hyp} \approx 5,285.9 \text{ ft}$$

Second triangle:

$$2 \text{ miles} \cdot 5,280 \text{ ft/mi} = 10,560 \text{ ft}$$

$$(500\text{ ft})^2 + (10{,}560\text{ ft})^2 = \text{hyp}^2$$
$$250{,}000\text{ ft}^2 + 111{,}513{,}600\text{ ft}^2 = \text{hyp}^2$$
$$111{,}763{,}600\text{ ft}^2 = \text{hyp}^2$$
$$\text{hyp} = \sqrt{111{,}763{,}600\text{ ft}^2}$$
$$\text{hyp} \approx 10{,}571.8\text{ ft}$$

Third triangle:

$$4\text{ miles} \cdot 5{,}280\text{ ft/mi} = 21{,}120\text{ ft}$$
$$(1{,}000\text{ ft})^2 + (21{,}120\text{ ft})^2 = \text{hyp}^2$$
$$1{,}000{,}000\text{ ft}^2 + 446{,}054{,}400\text{ ft}^2 = \text{hyp}^2$$
$$447{,}054{,}400\text{ ft}^2 = \text{hyp}^2$$
$$\text{hyp} = \sqrt{447{,}054{,}400\text{ ft}^2}$$
$$\text{hyp} \approx 21{,}143.7\text{ ft}$$

Lesson 2.15 Shortest Distance

Skills:

1.

$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(3-2)^2 + (18-6)^2} \\ &= \sqrt{1^2 + 12^2} \\ &= \sqrt{1+144} \\ &= \sqrt{145} \\ &\approx 12.0 \end{aligned}$$

2.

$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(1--1)^2 + (-12-12)^2} \\ &= \sqrt{(1+1)^2 + (-12-12)^2} \\ &= \sqrt{2^2 + (-24)^2} \\ &= \sqrt{4+576} \\ &= \sqrt{580} \\ &\approx 24.1 \end{aligned}$$

Concepts and Applications:

3. Student #1: This student forgot the square root.
Student #2: This student did not handle the negative number correctly when he subtracted the x-values. It should be $2-(-1)$, not $2-1$.
Student #3: This student did not do the operations in the correct order. He needed to square and add before taking the square root.

$$\begin{aligned} d &= \sqrt{(x_2-x_1)^2+(y_2-y_1)^2} \\ &= \sqrt{(-1-2)^2+(16-18)^2} \\ &= \sqrt{(-3)^2+(-2)^2} \\ &= \sqrt{9+4} \\ &= \sqrt{13} \\ &\approx 3.6 \end{aligned}$$

4. a.

$$\begin{aligned} d &= \sqrt{(x_2-x_1)^2+(y_2-y_1)^2} \\ &= \sqrt{(4--2)^2+(13-5)^2} \\ &= \sqrt{6^2+8^2} \\ &= \sqrt{36+64} \\ &= \sqrt{100} \\ &= 10 \end{aligned}$$

b.

$$\begin{aligned} d &= \sqrt{(x_2-x_1)^2+(y_2-y_1)^2} \\ &= \sqrt{(-2-4)^2+(5-13)^2} \\ &= \sqrt{(-6)^2+(-8)^2} \\ &= \sqrt{36+64} \\ &= \sqrt{100} \\ &= 10 \end{aligned}$$

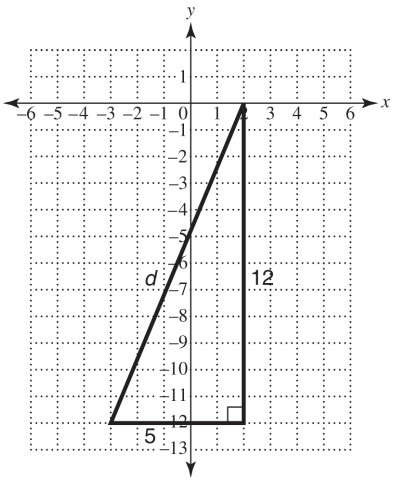
c. When you square the differences in the distance formula, you get the same positive number regardless of the order of the subtraction. Changing the order of the subtraction changes the sign on the difference but does not affect the square of the difference.

5. The order of the subtraction does not matter in the distance formula, and the same answer will be obtained regardless of the order of the subtraction. The *x*-values do not have to be subtracted in the same order as the *y*-values to obtain the correct answer. The order of the subtraction does not matter in the slope formula, but the same order must be used in the numerator and denominator of the slope fraction.

Cycle 2 Part 2 Recap

Skills:

1.



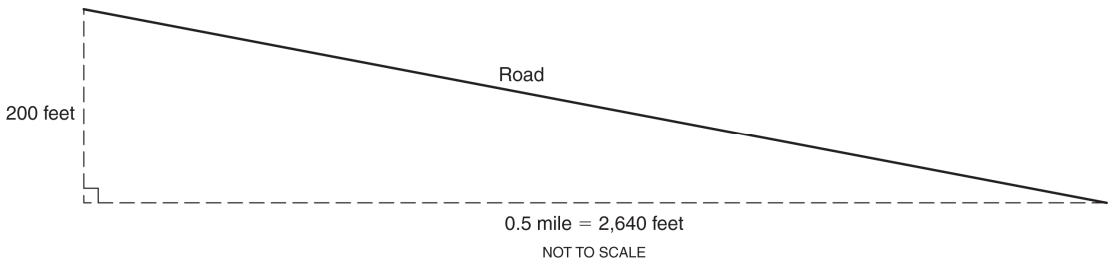
$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(2 - -3)^2 + (0 - -12)^2}$$
$$= \sqrt{(2+3)^2 + (0+12)^2}$$
$$= \sqrt{5^2 + 12^2}$$
$$= \sqrt{25 + 144}$$
$$= \sqrt{169}$$
$$= 13$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{0 - (-12)}{2 - (-3)}$$
$$= \frac{0 + 12}{2 + 3}$$
$$= \frac{12}{5}$$

Concepts and Applications

2. $\text{slope} = \dfrac{\text{vertical distance}}{\text{horizontal distance}}$ $\quad \text{distance} = \sqrt{(\text{horizontal distance})^2 + (\text{vertical distance})^2}$

3.



$$\begin{aligned}\text{grade} &= \frac{\text{rise}}{\text{run}} \\ &= \frac{-200 \text{ ft}}{2,640 \text{ ft}} \\ &\approx 0.076 = 7.6\%\end{aligned}$$

$$\begin{aligned}\text{length of road} &= \sqrt{(\text{horizontal distance})^2 + (\text{vertical distance})^2} \\ &= \sqrt{200^2 + 2,640^2} \\ &= \sqrt{40,000 + 6,969,600} \\ &= \sqrt{7,009,600} \\ &\approx 2,647.6 \text{ ft}\end{aligned}$$

Cycle 2: Why Does It Matter?

Part 3

Lesson 2.17 Parts of Speech

Skills:

1. The multiplication in the numerator of the fraction should be performed first.

2. The squaring of the *x*-value under the square root should be done first.

Concepts and Applications:

3. a. 4
   b. $4(x+y)$
   c. x
   d. $\frac{x}{4}$

4. Jake is incorrect. The addition must be performed before the square root since the square root functions as a grouping symbol. The expression is correctly simplified as follows:

$$\sqrt{25+9} = \sqrt{34} \approx 5.83$$

5. a.

x	y	$\sqrt{x+y}$	$\sqrt{x}+\sqrt{y}$	Are the expressions equal?
0	0	0	0	Yes
1	1	1.41	2	No
0	1	1	1	Yes
16	4	4.47	6	No
4	0	2	2	Yes
25	9	5.83	8	No

   b. Yes. Since there is at least one example in the table for which the expressions are not equal, that is enough to disprove the claim that $\sqrt{x+y} = \sqrt{x}+\sqrt{y}$.
   c. No. Even though there is at least one example in the table for which the expressions are equal, that is not enough to prove the claim that $\sqrt{x+y} = \sqrt{x}+\sqrt{y}$.
   d. No. There are examples in the table for which the two expressions are not equal.
   e. No. There are examples in the table for which the two expressions are not equal.

6. No. The expressions are not equivalent since, for example, $(5-2)^2 = 3^2 = 9$ but $5^2 - 2^2 = 25 - 4 = 21$. This one counterexample shows that the expressions are not equivalent.

$$\begin{aligned}(a-b)^2 &= (a-b)(a-b)\\ &= a(a-b)-b(a-b)\\ &= a^2-ab-ab+b^2\\ &= a^2-2ab+b^2\end{aligned}$$

7. a.
$$\begin{aligned}3n^2-2 &= 3\cdot 5^2-2\\ &= 3\cdot 25-2\\ &= 75-2\\ &= 73\end{aligned}$$
b. square, multiply by 3, subtract 2

8. a.
$$\begin{aligned}\frac{2(x-1)}{3} &= \frac{2(10-1)}{3}\\ &= \frac{2(9)}{3}\\ &= \frac{18}{3}\\ &= 6\end{aligned}$$
b. subtract 1, multiply by 2, divide by 3 or subtract 1, divide by 3, multiply by 2.

Lesson 2.18 In the Swing of Things

Skills:

1.
$$\begin{aligned}G &= \frac{1}{\sqrt{2+x}}\\ &= \frac{1}{\sqrt{2+7}}\\ &= \frac{1}{\sqrt{9}}\\ &= \frac{1}{3}\end{aligned}$$

Concepts and Applications:

2. a. square, then multiply by π
b.

r (inches)	A (square inches)
12	452.39
13	530.93
14	615.75

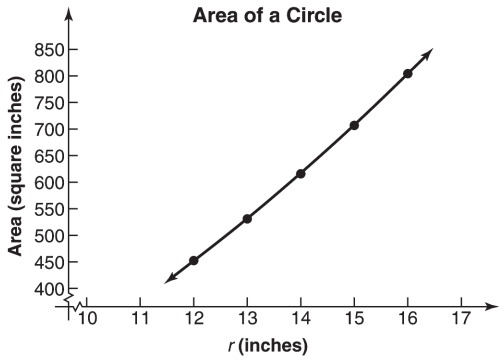
15	706.86
16	804.25

c. square inches

d. The formula represents a function since each value of the radius is matched with exactly one value of the area. The formula does not represent a linear function since the area does not increase by the same amount each time the radius goes up by one inch.

e. The radius would need to be a little less than 15 inches to yield an area of 700 square inches.

f.



g. The data points might look like they fall in a line, depending on the scale used for the graph. We can be sure the data is not linear, however, by analyzing the table and noting that the area values do not increase by the same amount each time the radius increases by one inch.

Lesson 2.19 Error and Estimation: Rounding

Skills:

1.

$$\begin{aligned}\text{APY} &= \left(1+\frac{r}{n}\right)^{n}-1\\ &= \left(1+\frac{0.04}{12}\right)^{12}-1\\ &\approx 0.0407 = 4.07\%\end{aligned}$$

Concepts and Applications:

2. a. divide i by n, add 1, raise to the $\frac{n}{p}$ power, subtract 1

b.

$$\begin{aligned}r &= \left(1+\frac{i}{n}\right)^{\frac{n}{p}}-1\\ &= \left(1+\frac{0.075}{2}\right)^{\frac{2}{12}}-1\\ &\approx 0.006155 = 0.6155\%\end{aligned}$$

c.

$$r = \left(1 + \frac{i}{n}\right)^{\frac{n}{p}} - 1$$
$$= \left(1 + \frac{0.0825}{12}\right)^{\frac{12}{24}} - 1$$
$$\approx 0.003432 = 0.3432\%$$

Cycle 2 Wrap-Up: Cycle 2 Profile

Write the slope of the line in two ways: $m = \frac{a}{b}$; $m = \frac{y_2 - y_1}{x_2 - x_1}$

Write two statements about the value of *d*: $d = \sqrt{a^2 + b^2}$; $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Snapshots:

Commutative
Associative
Distributive

+	Positive	Negative
Positive	+	depends
Negative	depends	−

×	Positive	Negative
Positive	+	−
Negative	−	+

$x^a \cdot x^b = x^{a+b}$
$(x^a)^b = x^{ab}$

Cycle 2 Wrap-Up: Vocabulary Check

1. negative number
2. integers
3. opposite
4. absolute value
5. real numbers
6. mean
7. exponent, base
8. like terms, coefficients
9. polynomial
10. monomial, binomial, trinomial
11. degree
12. PEMDAS
13. commutative property
14. associative property
15. distributive property

16. Pythagorean theorem
17. hypotenuse, legs
18. Pythagorean triple
19. slope
20. distance formula
21. operation, objects
22. radical function

Cycle 2 Wrap-Up: Concepts and Applications Review

1.

$$-8+(-8)=-16$$
$$-8-(-8)=-8+8=0$$
$$-8\times(-8)=64$$
$$-8\div(-8)=1$$

2. a.

-2

$-2-1=-3$

$-3\times 3=-9$

$-9+6=-3$

$-3\div 3=-1$

$-1-(-2)=-1+2=1$

$\frac{1}{2}$

$\frac{1}{2}-1=-\frac{1}{2}$

$-\frac{1}{2}\times 3=-\frac{3}{2}$

$-\frac{3}{2}+6=-\frac{3}{2}+\frac{12}{2}=\frac{9}{2}$

$\frac{9}{2}\div 3=\frac{9}{2}\div\frac{3}{1}=\frac{\cancel{9}^{3}}{2}\times\frac{1}{\cancel{3}_{1}}=\frac{3}{2}$

$\frac{3}{2}-\frac{1}{2}=\frac{1}{2}$

b. $\frac{3(x-1)+6}{3}-x$

c. $\frac{3(x-1)+6}{3}-x=\frac{3x-3+6}{3}-x=\frac{\cancel{3}^{1}x+\cancel{1}^{1}}{3}-x=x+1-x=1$

3. a. Friday: $\$200-\$100-\$150-\$15-\$20-\$20=-\$105$;
$-\$105-\$20$ (overdraft fees of \$5 per item) $=-\$125$

Saturday: $-\$125-\$10-\$35=-\$170$;
$-\$170-\$10$ (overdraft fees of \$5 per item) $-\$5$ (overdraft fee of \$5 per day)
$=-\$185$

Sunday: $-\$185 - \$85 = -270$

$-\$270 - \$5$ (overdraft fees of \$5 per item) $- \$5$ (overdraft fee of \$5 per day)

$= -\$280$

b. It would be better if the smaller charges cleared first on Friday. Since there is a charge based on the number of items once the account is overdrawn, the fee will be less if most of the smaller items clear first. If the book store charge was processed last, it would be the only one to incur a fee on Friday since the other items total to \$155 and would be covered by the \$200 balance.

c. No. If you transfer \$150 first thing Saturday, the balance will only be \$25 and insufficient to cover both charges on Saturday. Transferring \$150 on Saturday morning would avoid one of the overdraft fees on Saturday though.

4. a. $\dfrac{72+78+75+8}{4} = 58.25$

b. $\dfrac{72+78+75+82}{4} = 76.75$

c. $\dfrac{58.25-76.75}{76.75} \approx -0.24 = -24\%$

5.

$$\begin{aligned} S &= 2\pi r^2 + 2\pi rh \\ &= 2\pi(2 \text{ ft})^2 + 2\pi(2 \text{ ft})(10 \text{ ft}) \\ &= 8\pi \text{ ft}^2 + 40\pi \text{ ft}^2 \\ &= 48\pi \text{ ft}^2 \\ &\approx 150.8 \text{ ft}^2 \end{aligned}$$

6. a. Yes, but with more than one variable.

b. $3x^2$ and $-x^2$ are like terms. $-14x, 5x,$ and $9x$ are like terms.

c. $\underline{3x^2} \underline{-14x} + 3y^3 - 2y \underline{+5x} \underline{-x^2} \underline{+9x} - 1 = 2x^2 + 3y^3 - 2y - 1$

7. a. $(5-14)(2-4) = (-9)(-2) = 18$

b.

$$\begin{aligned} (5-14)(2-4) &= (-9)(2-4) \\ &= -9 \cdot 2 - 9 \cdot -4 \\ &= -18 + 36 \\ &= 18 \end{aligned}$$

c.

$$\begin{aligned}(5-14)(2-4) &= 5(2-4)-14(2-4)\\ &= 5\cdot 2+5\cdot -4-14\cdot 2-14\cdot -4\\ &= 10-20-28+56\\ &= 18\end{aligned}$$

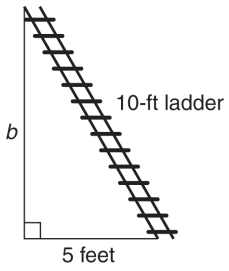
8.

$$\begin{aligned}99(25) &= (100-1)(25)\\ &= 100\cdot 25-1\cdot 25\\ &= 2,500-25\\ &= 2,475\end{aligned}$$

9.

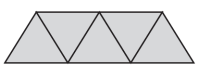
$$\begin{aligned}18^2+18^2 &= \text{hyp}^2\\ 324+324 &= \text{hyp}^2\\ 648 &= \text{hyp}^2\\ \sqrt{648} &= \text{hyp}\\ \text{hyp} &\approx 25.5 \text{ in.}\end{aligned}$$

10.



$$\begin{aligned}5^2+b^2 &= 10^2\\ 25+b^2 &= 100\\ b^2 &= 75\\ b &= \sqrt{75}\\ b &\approx 8.7 \text{ ft}\end{aligned}$$

11. a.



7 chairs

b. $n+2$

c. There is one seat for each table, plus two more seats on the ends.

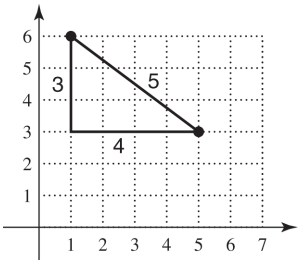
12. First, find the difference of the *x*-coordinates and the difference of the *y*-coordinates.
Second, square each of those differences.
Third, add the squared differences.
Fourth, take the square root of that sum.

13. First, subtract the *y*-values.
Second, subtract the *x*-values in the same order.
Third, divide the difference in the *y*-values by the difference in the *x*-values.

14.

Expression	Translation
$\sqrt{x}+2$	Take the square root of *x* and then add 2.
$\sqrt{x+2}$	Take the square root of the quantity *x* plus 2.
$\frac{5}{x+2}$	Divide 5 by the sum of *x* and 2.
$\frac{5}{x}+2$	Divide 5 by *x* and then add 2.
$2x^2$	Square x and then multiply the result by 2
$(2x)^2$	Square the product of 2 and *x*.
$\lvert x-1\rvert$	Take the absolute value of the difference of *x* and 1.
$\lvert x\rvert-1$	Take the absolute value of *x* and then subtract 1.

15. Answers will vary. One possible answer is ($(1,6)$ and $(5,3)$.



16. a. $$\frac{57+65.5+72+71+64.25+69.5+58.75+57.75+69.25+64}{10}=\frac{649}{10}=64.9 \text{ inches}$$

b.
$$C=2\pi r=\pi d$$
$$64.9 \text{ in.}=\pi d$$
$$d=\frac{64.9 \text{ in.}}{\pi}\approx 20.7 \text{ in.}$$

c. Take each circumference and divide by π to produce the following set of diameters (each rounded to the nearest hundredth)

18.14 20.85 22.92 22.60 20.45 22.12 18.70 18.38 22.04 20.37

Average diameter:

$$\frac{18.14+20.85+22.92+\ldots+20.38}{10}=\frac{206.66}{10}\approx 20.7 \text{ in.}$$

This produces the same result.

17. A steep line would have a large, positive value for the slope. Since there is no largest positive number, there is no steepest line. For any line, you can always draw one that is a little steeper because for any positive slope you can always find a slightly larger positive number. A vertical line may seem to be the steepest line, but by definition, it does not have a slope.